This document details the algorithms and formulas used to develop the PSLoss.xlsb Excel spreadsheet, available for download at www.yakpol.net.

These calculations are presented using Mathcad Prime for maximum clarity and transparency.

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Introduction:

This document explains the computational algorithm used in the PSLoss spreadsheet to define prestressing tendon configurations with multiple parabolic and straight segments and to calculate anchor set and friction losses along the tendon, as well as elongations at the stressing end. The algorithm follows the AASHTO LRFD provisions for stress losses due to friction and curvature changes and uses U.S. customary units. The example illustrates a tendon stressed first from the left end and then from the right.

Input

$f_{pu} := 270 \cdot ksi$
$E_s := 28500 \cdot ksi$
$\mu := 0.25$
$k := 0.0002 \cdot ft^{-1}$
$\Delta S := 0.375 \cdot in$

Define Tendon Profile

The tendon profile consists of parabolic and tangent segments. Each parabolic segment runs from an inflection point to a low or high point. Enter vertical coordinates only for low and high points; at inflection points they are calculated automatically, so you may enter 0.0. The program does not support two consecutive tangent segments. Use tangent segments only at tendon ends and to connect parabolic segments.

Jacking stress at **1st Pull** (from the left end)

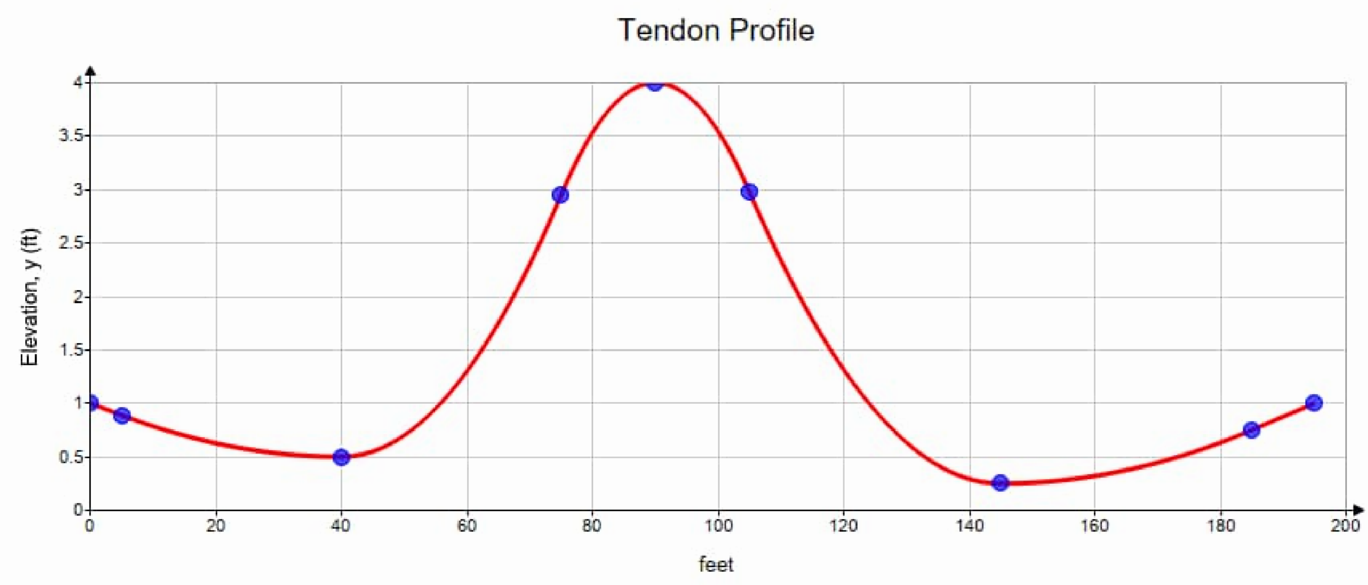
$pr1 := 0.75$ $F_{jack1} := pr1 \cdot f_{pu} = 202.5\ ksi$

Jacking stress at **2nd Pull** (from the right end)

$pr2 := 0.75$ $F_{jack2} := pr2 \cdot f_{pu} = 202.5\ ksi$

Length of tendon $Lt := x\_end_{\text{last}(x\_end)} = 195\ ft$

Nseg	Segment	x_end (ft)	y_end (ft)
“”	“Left Anchor”	0	1
0	Tangent	5	0
1	Sag falling	40	0.5
2	Sag rising	75	0
3	Crown rising	90	4
4	Crown falling	105	0
5	Sag falling	145	0.25
6	Sag rising	185	0
7	Tangent	195	1
8	Empty		
9	Empty		
10	Empty		
11	Empty		
12	Empty		



Functions to find stresses and friction losses

Initial values of anchor set stress loss and the convergence length:
(may need to alternate if app cannot solve equations)

$\Delta F := 10 \cdot ksi$
$x_c := 30 \cdot ft$

For 1st Pull from the left anchor:

Angle change from Jacking End: $f\theta(x) := \text{linterp}(sta, \theta, x)$
Losses from Jacking end: $df(Fj, x) := Fj \cdot (1 - \exp(-\mu \cdot f\theta(x) - k \cdot x))$

Stress before anchor set: $f_{bas}(Fj, x) := Fj - df(Fj, x)$
Stress after anchor set: $f_{aas}(\Delta F, Fj, x) := Fj - \Delta F + df(Fj, x)$
ΔF - Stress loss at the anchor due to anchor set. To be found iteratively by setting the area between functions for stresses before and after losses to $\Delta S \cdot E_s$.

Anchor set influence length:

$$Linf(\Delta F, Fj, x) := \textbf{root}\left(\text{linterp}\left(x, f_{bas}(Fj, x), x_c\right) - \text{linterp}\left(x, f_{aas}(\Delta F, Fj, x), x_c\right), x_c\right)$$

Area = $\Delta S \cdot E_s$

$$solution(\Delta F, Fj, x) := \textbf{root}\left(\int_{0 \cdot ft}^{min(Linf(\Delta F, Fj, x), Lt)} \left(f_{bas}(Fj, x) - f_{aas}(\Delta F, Fj, x)\right) \mathrm{d}x - \Delta S \cdot E_s, \Delta F\right)$$

Find stresses at 1st Pull (from the Left end)

$pr1 = 0.75$ $F_{jack1} = 202.5$ ***ksi***

Stress loss at anchor: $\Delta F1 := solution(\Delta F, F_{jack1}, sta) = 19.571$ ***ksi***

Anchor set influence length: $xc1 := Linf(\Delta F1, F_{jack1}, sta) = 70.228$ ***ft***

Elongation at left anchor: $elong1 := \frac{1}{E_s} \cdot \int_{0 \cdot ft}^{Lt} (f_{bas}(F_{jack1}, x)) \, dx = 15.095$ ***in***

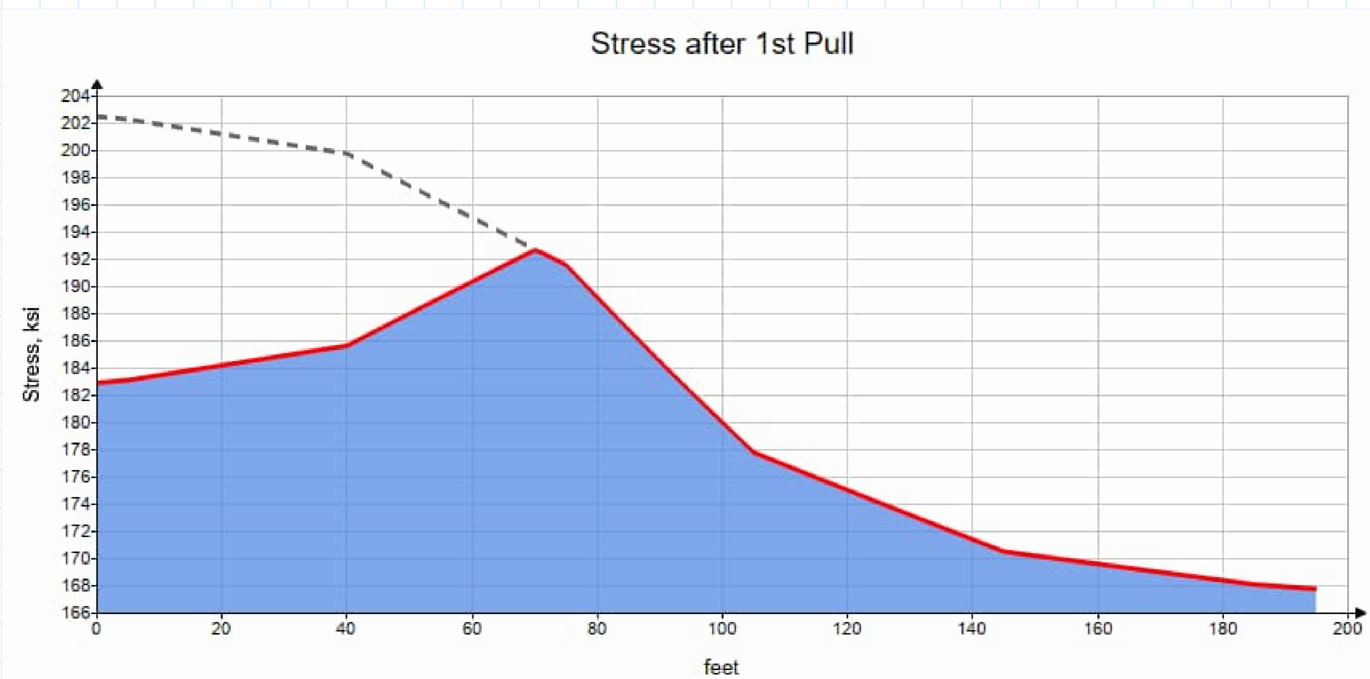
Stress after anchor set $f1a(x) := \text{if}(x < xc1, f_{aas}(\Delta F1, F_{jack1}, x), f_{bas}(F_{jack1}, x))$

Stress before anchor set $f1b(x) := f_{bas}(F_{jack1}, x)$

Stress at anchor $F_{anchor} := f1a(0 \cdot ft) = 182.929$ ***ksi*** $\frac{F_{anchor}}{f_{pu}} = 0.678$

Maximum stress $F_{max} := f1a(xc1) = 192.695$ ***ksi*** $\frac{F_{max}}{f_{pu}} = 0.714$

$x2 := 0 \cdot ft, 1 \cdot ft .. xc1$



Find stresses at 2nd Pull (from the Right end)

$pr2 = 0.75$ $F_{jack2} = 202.5$ ***ksi***

Reverse initial arrays for coordinates and slope change. The coordinate X=0 at pulling end.

$rx := \text{sort}\left(sta_{\text{last}(sta)} - sta\right)$ $\qquad r\theta := \text{sort}\left(\theta_{\text{last}(\theta)} - \theta\right)$

Redefine earlier introduced functions to update for rx and $r\theta$ (which were not used as arguments)

$f\theta(x) := \text{linterp}(rx, r\theta, x)$

$df(Fj, x) := Fj \cdot (1 - \exp(-\mu \cdot f\theta(x) - k \cdot x))$

Stress before anchor set: $f_{bas}(Fj, x) := Fj - df(Fj, x)$

Stress after anchor set: $f_{aas}(\Delta F, Fj, x) := Fj - \Delta F + df(Fj, x)$

$Linf(\Delta F, Fj, x) := \textbf{root}\left(\text{linterp}\left(x, f_{bas}(Fj, x), x_c\right) - \text{linterp}\left(x, f_{aas}(\Delta F, Fj, x), x_c\right), x_c\right)$

$$solution(\Delta F, Fj, x) := \textbf{root}\left(\int_{0 \cdot ft}^{min(Linf(\Delta F, Fj, x), Lt)} \left(f_{bas}(Fj, x) - f_{aas}(\Delta F, Fj, x)\right) \mathrm{d}x - \Delta S \cdot E_s, \Delta F\right)$$

Prestress loss at 2nd anchor: $\Delta F2 := solution\left(\Delta F, F_{jack2}, rx\right) = 17.774\ ksi$

Anchor set Influence zone length (from right): $xc2 := Linf\left(\Delta F2, F_{jack2}, rx\right) = 77.523\ ft$

Check if Pull 2 was beneficial. Stress at right end:

Before Pull 2: $f1a(Lt) = 167.775\ ksi$

After Pull 2: $F_{jack2} - \Delta F2 = 184.726\ ksi$

$Check\_Pull2\_jacking\left(F_{jack2} - \Delta F2 \geq f1a(Lt)\right) = \text{"Good"}$

Further calculations are true only for $F_{jack2} - \Delta F2 \geq f1a(Lt) = 1$ condition.

Stress after anchor set

Find intersection of stresses for pull 1 after anchor set and pull 2 before anchor set

$xc := 90 \cdot ft$ Initial value for solving. Measured from left end.

if xc3 does not converge, consult the chart to provide a close initial value (xc) for intersection of the functions.

The other possible problem is when stress at the left anchor after 2nd pull is higher than at the first pull (after anchor set). Then $xc3 = 0$

$f1a(0 \cdot ft) = 182.929\ ksi$

$f_{bas}\left(F_{jack2}, Lt\right) = 167.775\ ksi$

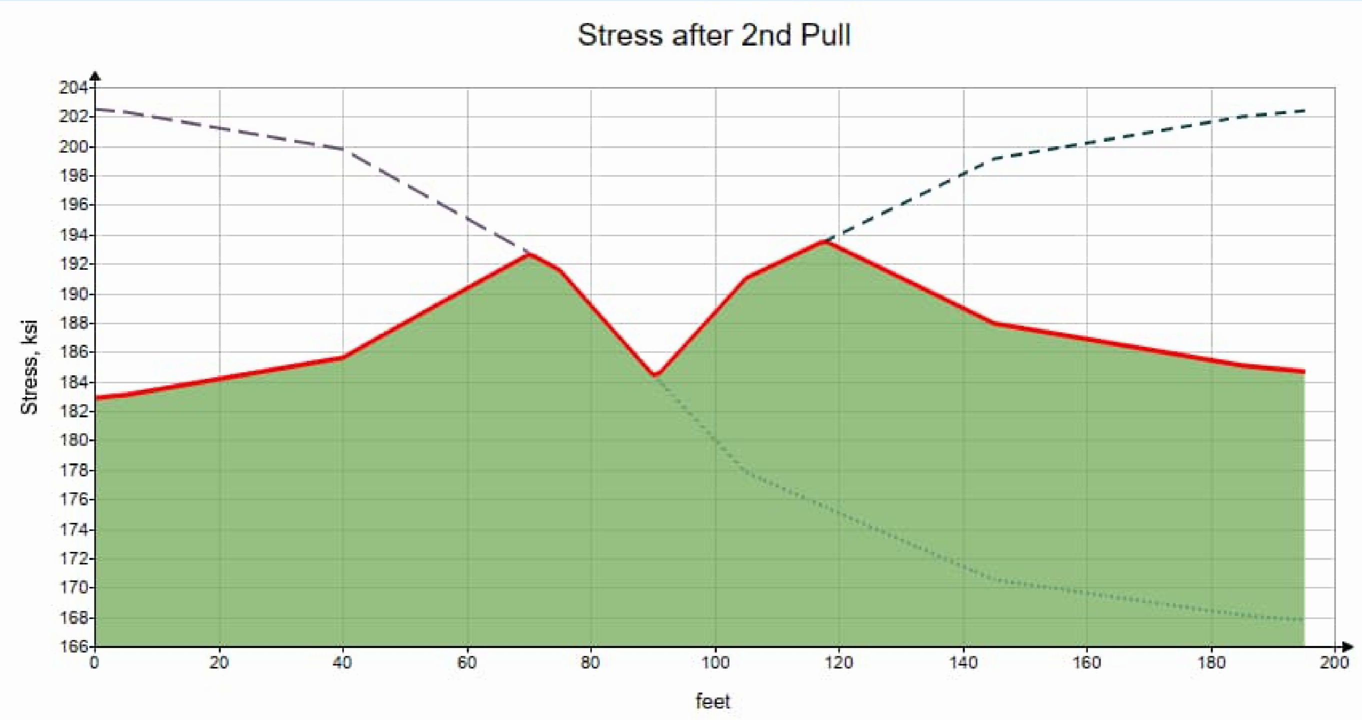
$xc3 := \textbf{if}\left(f_{bas}\left(F_{jack2}, Lt\right) \geq f1a(0 \cdot ft), 0 \cdot ft, \textbf{root}\left(f1a(xc) - f_{bas}\left(F_{jack2}, Lt - xc\right), xc\right)\right) = 90.388\ ft$

Elongation at 2nd Pull: $$elong2 := \frac{1}{E_s} \cdot \int_{\max(xc3, 0 \cdot ft)}^{Lt} \left(f_{bas}\left(F_{jack2}, Lt - x\right) - f1a(x)\right) \mathrm{d}x = 1.07\ in$$

Stresses after anchor set:

$$f2a(x) := \begin{bmatrix} 0 \cdot ft \le x \le xc3 \\ xc3 < x \le Lt - xc2 \\ x > Lt - xc2 \end{bmatrix} \cdot \begin{bmatrix} f1a(x) \\ f_{bas}(F_{jack2}, Lt - x) \\ f_{aas}(\Delta F2, F_{jack2}, Lt - x) \end{bmatrix}$$

$$x3 := 0 \cdot ft, 1 \cdot ft .. xc2$$



Summary of Analysis

Pull 1 $pr1 = 0.75$

Elongation $elong1 = 15.1\ in$

$$\begin{bmatrix} \text{“Jacking Stress”} \\ \text{“Stress at anchor after anchor set”} \\ \text{“Maximum stress”} \\ \text{“Stress at the 2nd end”} \end{bmatrix} \begin{bmatrix} F_{jack1} \\ f1a(0 \cdot ft) \\ f1a(xc1) \\ f1a(Lt) \end{bmatrix} = \begin{bmatrix} 202.5 \\ 182.93 \\ 192.69 \\ 167.78 \end{bmatrix} ksi$$

Pull 2 $pr2 = 0.75$

Elongation $elong2 = 1.07\ in$

$$\begin{bmatrix} \text{“Jacking Stress”} \\ \text{“Stress at anchor after anchor set”} \\ \text{“Maximum stress”} \\ \text{“Stress at conversion of 1 and 2 pull”} \end{bmatrix} \begin{bmatrix} F_{jack2} \\ f2a(Lt) \\ f2a(Lt - xc2) \\ f2a(xc3) \end{bmatrix} = \begin{bmatrix} 202.5 \\ 184.73 \\ 193.58 \\ 184.32 \end{bmatrix} ksi$$

Comment: See the printout of PSLoss Spreadsheet solving the same problem.
The results are nearly identical at 99.8% match.

Spreadsheets for Structural Engineers	Project: Mathcad Example	Engineer: YP Date: 14-Jan-26
PSLoss	Subject: Algorith Verification with Mathcad	Checker: Date:

9/26/2017

PRESTRESS LOSSES IN POST-TENSIONING TENDON

Units: US

Ultimate strength of strand	f_{pu} =	270	ksi
Modulus of elasticity of strand	E_s =	28500	ksi
Cross-sectional area of strand		0.217	in²
Number of strands in tendon		12	
Cross-sectional area of tendon		2.604	in²
Tendon length		195	ft
Coefficient of friction	μ =	0.25	
Wobble Friction Coefficient	k =	0.0002	1/ft
Anchor set	ΔS =	0.375	in
Additional prestress losses	Δf_{LT} =	30	ksi (long-term and elastic shortening)

Friction losses formula style
AASHTO
$\Delta F = F_{jack}\ [1 - e\verb|^| -(\mu\theta + kx)]$

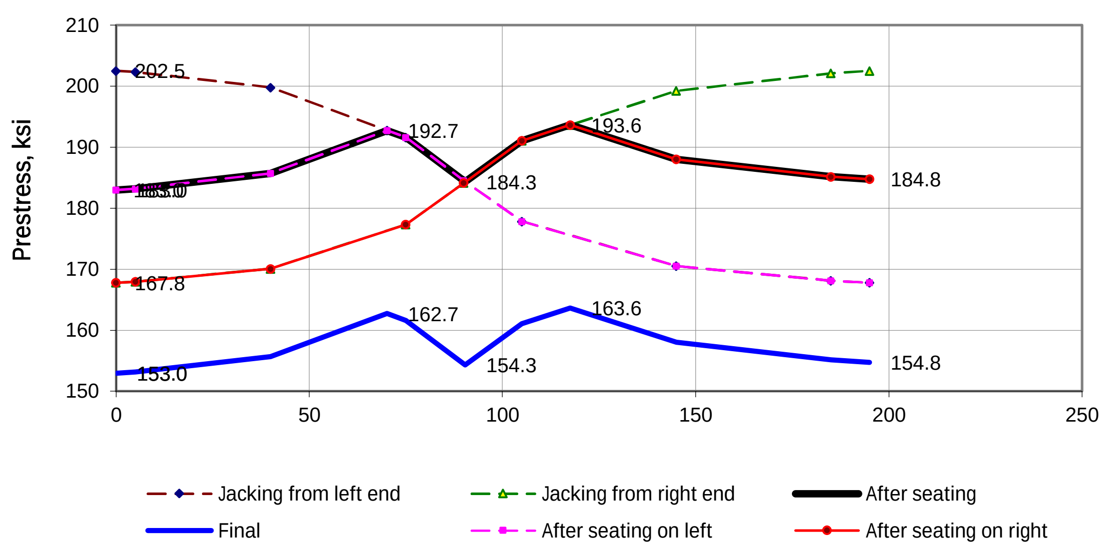
		Left Pull	Right Pull
Stressing ratio	F_{Jack}/f_{pu} =	0.75	0.75
Stressing order		1	2

<u>Summary of Results</u>

Average stress after seating	187.73	ksi
Final average stress	157.73	ksi
Final average force in tendon	410.72	Kip

		Left Pull	Right Pull	
Jacking stress	F_{Jack} =	202.5	202.5	ksi
Jacking force	P_{Jack} =	527.31	527.31	Kip
Anchor set influence zone	L_c =	70.16	77.46	ft
Prestress losses at anchor	ΔF =	19.54	17.75	ksi
Stress at anchor	f_{anchor} =	182.96	184.75	ksi
Stress at end of inluence zone		192.73	193.63	ksi
Elongation before anchor set		15.10	1.07	in

Stress ratio at anchorage	0.684	fpu
Max. stress ratio along tendon	0.717	fpu



Spreadsheets for Structural Engineers	Project: Mathcad Example	Engineer: YP Date: 14-Jan-26
PSLoss	Subject: Algorith Verification with Mathcad	Checker: Date:

9/26/2017

PRESTRESS LOSSES IN POST-TENSIONING TENDON

Distance from left end	Segment Length	Angular Change per Segment		
		Vertical Profile	Horizontal Profile	Total accumulative
ft	ft	rad	rad	rad
0	0			
5	5	0.00000	0.00000	0.0000
40	35	0.02222	0.00000	0.0222
75	35	0.13910	0.00000	0.1613
90	15	0.13910	0.00000	0.3004
105	15	0.13553	0.00000	0.4359
145	40	0.13553	0.00000	0.5715
185	40	0.02499	0.00000	0.5965
195	10	0.00000	0.00000	0.5965

Distance from left end	Stress after anchor set	Stress after additional losses
ft	ksi	ksi
0.00	183.0	153.0
5.00	183.2	153.2
40.00	185.7	155.7
70.16	192.7	162.7
75.00	191.6	161.6
90.00	184.5	154.5
90.39	184.3	154.3
105.00	191.1	161.1
117.54	193.6	163.6
145.00	188.0	158.0
185.00	185.2	155.2
195.00	184.8	154.8

Spreadsheets for Structural Engineers	Project: Mathcad Example	Engineer: YP Date: 14-Jan-26
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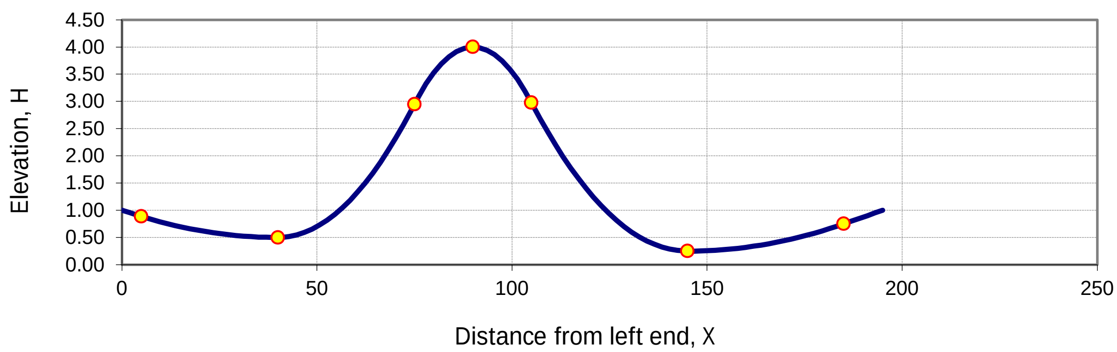
TENDON PROFILE



All dimensions are **ft** Web slope H/V = 0 Angle correction factor = 1.000

Vertical Profile

Segment type	ΔX	X1	X2	H1	H2
Tangent	5	0	5	1	0
Sag falling	35	5	40	0.88889	0.5
Sag rising	35	40	75	0.5	
Crown rising	15	75	90	2.95	4
Crown falling	15	90	105	4	
Sag falling	40	105	145	2.97727	0.25
Sag rising	40	145	185	0.25	
Tangent	10	185	195	0.75	1

Horizontal Profile

X	Segment type	Radius
0		
195	Tangent	

THEORY PRESTRESSING LOSSES DUE TO FRICTION AND ANCHOR SET

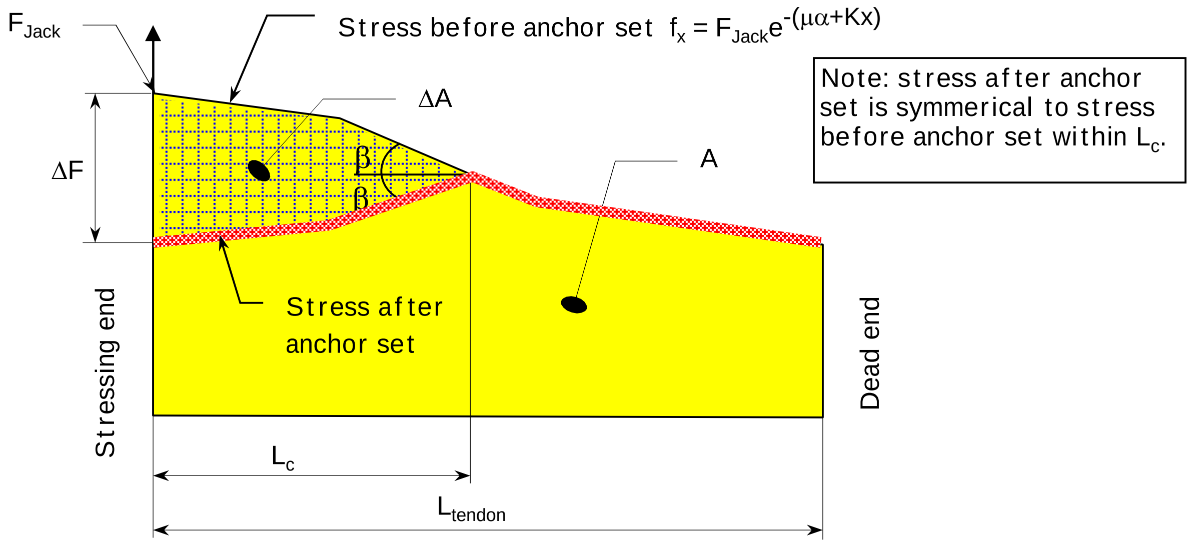


Fig. 1 Stressing from one end

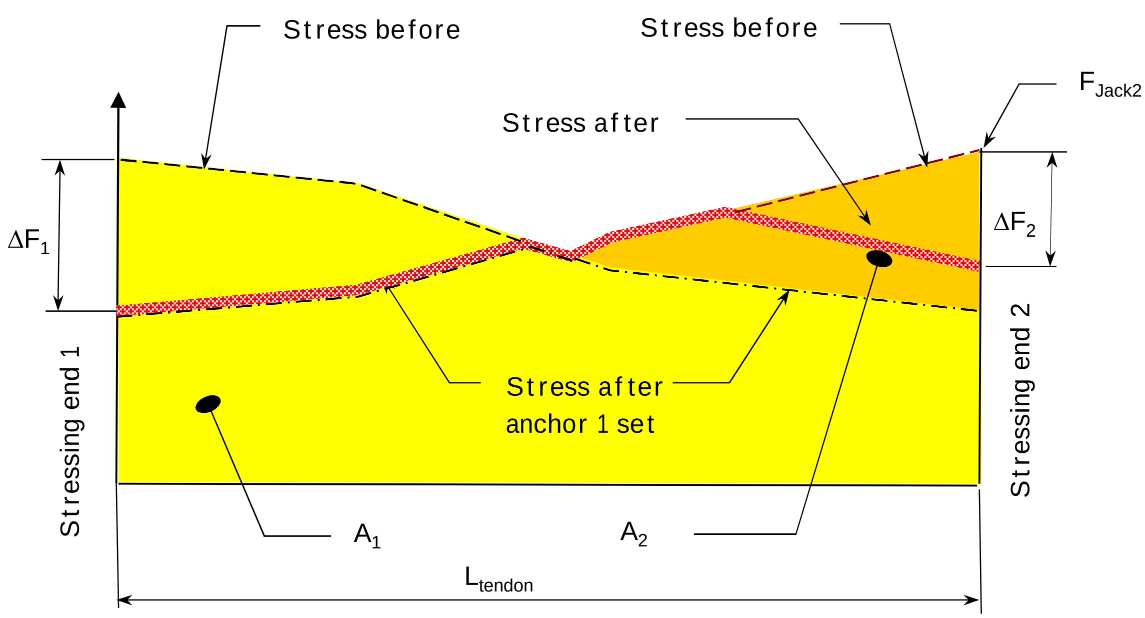


Fig. 2 Stressing from two ends

Stressing from one end (refer to Fig. 1 above)

$f_x = F_{Jack}e^{-(\mu\alpha+Kx)}$	Stress along the tendon at distance 'x' from the stressing end.
F_{Jack}	Jacking stress at anchor
α	Accumulative angle change from stressing end to 'x'
μ	Curvature Friction Coefficient
K	Wobble Friction Coefficient
$\Delta L = A/E_s$	Strand elongation during stressing operations.
E_s	Strand modulus of elasticity
A	Area below the graph of 'stress before anchor set' and above the line of zero stress (yellow colored area on Fig. 1)
$\Delta S = \Delta A/E_s$	Anchor set
ΔA	Area below graph 'stress before anchor set' and above graph 'stress after anchor set' (shaded area on Fig. 1)
Lc	Length of stress conversion zone.
ΔF	Prestress losses at anchor

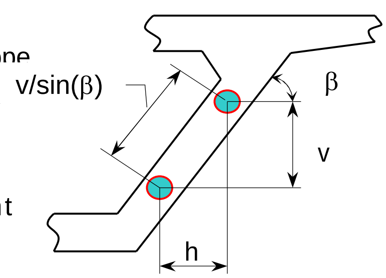
Stressing from two ends (refer to Fig. 2 above)

Compute stresses at second pull before and after anchor set independently from first pull. The resulting stress is the maximum of stresses due to first and second pulls (red line on Fig. 2)

$\Delta L_2 = A_2/E_s$	Elongations during second pull
A_2	Area below graph 'stress before anchor 2 set' and above graph 'stress after anchor 1 set' (orange shaded area on Fig. 2).

Angle change within a parabolic segment of tendon

$y = aX^2+bX+c$	Parabolic equation of tendon
$\alpha_1 = \text{atan}[(2aX_1+b)/\sin(\beta)]$	Start angle adjusted for web slope
$\alpha_2 = \text{atan}[(2aX_2+b)/\sin(\beta)]$	End angle adjusted for web slope
$\Delta\alpha = \text{abs}(\alpha_2-\alpha_1)$	Angle change within a parabolic segment
a, b, c	Parabolic coefficients
X_1, X_2	Start and End stations of parabolic segment
$\beta = \text{atan}(v/h)$	Web slope angle to horizon

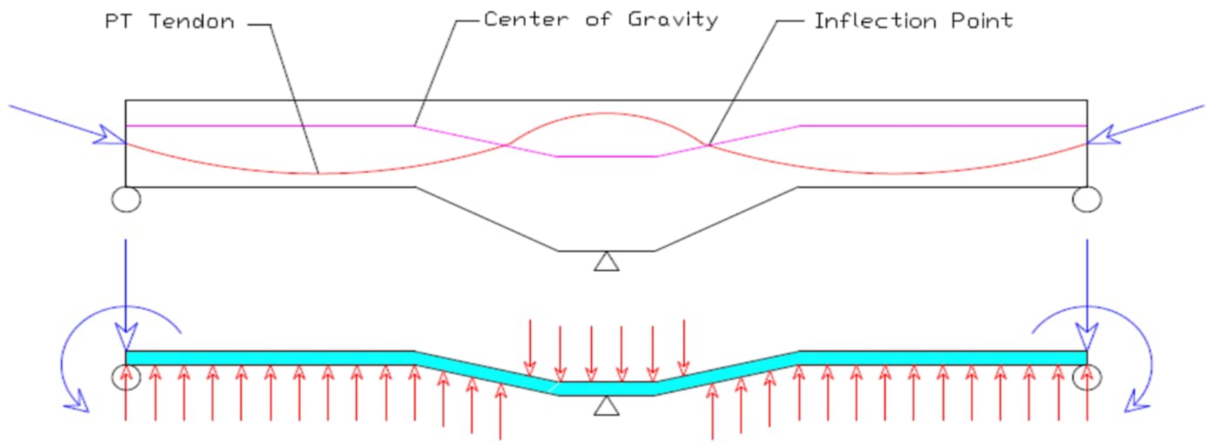


Combining tendon angular changes in vertical and horizontal planes

$\Delta\alpha$	Total angular change within a segment
$\Delta\alpha_v$, $\Delta\alpha_h$	Angular change in ortogonal planes
$\cos(\Delta\alpha)=\cos(\Delta\alpha_v)*\cos(\Delta\alpha_h)$	

Equivalent beam loads due to post-tensioning

PSLoss provides two methods of computing equivalent beam loads due to prestressing forces.
1. Nodal Loads method computes series of vertical concentrated loads applied at the beam nodes. Concentrated bending moments are applied at the location of anchorages. Total (primary and secondary) bending moments and shear forces and secondary axial forces due to prestressing can be calculated applying equivalent loads to structural model. Primary axial forces in concrete sections will be equal to prestressing forces in prestressing tendons. This method is preferable to the method of equivalent distributed loads which is not as easy to apply with variable elevation of the beam's centroid.



AASHTO LRFD recommendations for K (1/ft) and μ values in formula $\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx + \mu\alpha}\right.$

Table 5.9.5.2.2b-1—Friction Coefficients for Post-Tensioning Tendons

Type of Steel	Type of Duct	*K*	μ
Wire or strand	Rigid and semirigid galvanized metal sheathing	0.0002	0.15–0.25
	Polyethylene	0.0002	0.23
	Rigid steel pipe deviators for external tendons	0.0002	0.25
High-strength bars	Galvanized metal sheathing	0.0002	0.30

EC2 recommendations for K (1/m) and μ values in formula

$$\Delta P_{\mu}(x) = P_{max}(1 - e^{-\mu(\theta + kx)})$$ 0,005 < k < 0,01 per metre.

Table 5.1: Coefficients of friction μ of post-tensioned internal tendons and exte unbonded tendons

	Internal tendons [1)]	External unbonded tendons			
		Steel duct/ non lubricated	HDPE duct/ non lubricated	Steel duct/ lubricated	HDPE d lubricat
Cold drawn wire	0,17	0,25	0,14	0,18	0,12
Strand	0,19	0,24	0,12	0,16	0,10
Deformed bar	0,65	-	-	-	-
Smooth round bar	0,33	-	-	-	-

[1)] for tendons which fill about half of the duct

Note: HPDE - High density polyethylene

INSTRUCTIONS FOR USING PSLoss SPREADSHEET

Spreadsheet functionality:

- Computes friction and anchor set losses in prestressing tendon
- Computes tendon elongations
- Developes tendon vertical and horizontal profiles
- Calculates equivalent beam loads due to post-tensioning

Capabilities:

- Pulling from one or both ends
- Multiple angle changes in three-dimensional tendon profile
- SI and US measurement systems
- US and international codes

Instruction for data input

All input cells in the spreadsheet are formatted as a blue text on yellow background 4.35

The custom toolbar menu provides the following functions:

Calculate full	Updates cable angular changes with geometry described on worksheet 'Tendon Profile' and computes prestress losses
Calculate losses without updating cable profile	Calculates prestress losses without updating cable angular changes
Theory/Help	Activates current worksheet
Disclaimer	Displays PSLoss disclaimer

PSLoss Step by Step

1. Open 'Tendon Profile' worksheet (use menu Tendon Profile - Edit or click on 'Profile' tab below).
2. Establish tendon vertical profile built out of parabolic or tangent segments. Provides location of tangent and radial segments of horizontal profile.
3. On the worksheet 'Losses' enter required stressing data and click command 'Calculate full'
4. Review diagram of computed stresses along the tendon