Design of RC member for combined action of axial force, bending, shear and torsion in accordance with AASHTO LRFD 10th Edition

Introduction

This spreadsheet analyzes a reinforced concrete member of circular of rectangular cross-sections. subjected to combined axial load, flexure, shear, and torsion per AASHTO LRFD (10th Edition). The methodology follows strain compatibility principles and includes strength and serviceability checks.

Material properties

Strength: $\begin{bmatrix} f'_c \\ f_y \end{bmatrix} := \begin{bmatrix} 4 \\ 60 \end{bmatrix} \cdot ksi$

Max. concrete strain: $\varepsilon_{cu} := -0.003$

Concrete unit weight: $w_c := 0.145 \cdot kcf$ (for stiffness)

Modulus of elasticity:

Concrete $E_c := 120000 \cdot \left(\frac{w_c}{kcf}\right)^2 \cdot \left(\frac{f'_c}{ksi}\right)^{0.33} \cdot ksi = 3987\ ksi$

Steel $E_s := 29000 \cdot ksi$

Modular ratio $n := \frac{E_s}{E_c} = 7.27$

The analysis is performed using an equivalent rectangular compression block with niform compressive stress $\alpha_1 \cdot f'_c$ and a depth of $\beta_1 \cdot c$, where c is the distance to the Neutral Axis.

$$\alpha_1 := \max\left(0.75, 0.85 - \left(\frac{f'_c}{ksi} > 10\right) \cdot 0.02 \cdot \left(\frac{f'_c}{ksi} - 10\right)\right) = 0.85$$

$$\beta_1 := \max\left(0.65, 0.85 - \left(\frac{f'_c}{ksi} > 4\right) \cdot 0.05 \cdot \left(\frac{f'_c}{ksi} - 4\right)\right) = 0.85$$

Resistance factors

For combined axial force and bending moment analysis the resistance factor will be interpolated between $\phi_c := 0.75$ and $\phi_t := 0.9$ corresponding to compression and tension controlled strain limits in reinforcement per Section 5.6.2.1:

$$\varepsilon_{cl} := 0.002 + 0.002 \cdot min\left(1, \max\left(0, \frac{\frac{f_y}{ksi} - 60}{40}\right)\right) = 0.002$$

$$\varepsilon_{tl} := 0.005 + 0.003 \cdot min\left(1, \max\left(0, \frac{\frac{f_y}{ksi} - 75}{25}\right)\right) = 0.005$$

For shear $\phi_v := 0.9$

$SectionType :=$ CIRCULAR $= 1$

Rectangular section parameters $\text{if}(SectionType = 2, \text{"Applicable"}, \text{"N/A"}) = \text{"N/A"}$

Width $B := 36 \cdot in$
Height $H := 48 \cdot in$
Tie bar diameter $d_{tb} := 0.75 \cdot in$
Side clear to tie bar $clr_{tb} := 1.5 \cdot in$
Reinforcing layers
Distance from the bottom concrete face to c.g. of the layer - y_{s2}
Cross-sectional area of each bar in the layer - A_{sb}
Number of bars per layer - n_{b2}

y_{s2} (in)	Bar	n_{b2}
3	#6	5
13	#6	2
23	#6	2
33	#6	2
44	#9	7

$$A_{sb} := \overrightarrow{Asb(Bar)} = \begin{bmatrix} 0.44 \\ 0.44 \\ 0.44 \\ 0.44 \\ 1 \end{bmatrix} in^2 \quad d_{bo} := \overrightarrow{dbo(Bar)} = \begin{bmatrix} 0.83 \\ 0.83 \\ 0.83 \\ 0.83 \\ 1.24 \end{bmatrix} in$$

Reinforcing bar maximum overall diameter
$d_{b.max} := \max(d_{bo}) = 1.24\ in$

Distance between outer bars: $b_b := B - 2 \cdot (clr_{tb} + d_{tb} + 0.5 \cdot d_{b.max}) = 30.26\ in$
Number of bars at the bottom: $n_{b.bot} := \text{lookup}(\max(y_{s2}), y_{s2}, n_{b2})_{\text{ORIGIN}} = 7$

Number of bars at the top: $n_{b.top} := \text{lookup}(min(y_{s2}), y_{s2}, n_{b2})_{\text{ORIGIN}} = 5$

Spacing of outer bars: $\begin{bmatrix} s_{bot} \\ s_{top} \end{bmatrix} := b_b \div \begin{bmatrix} n_{b.bot} - 1 \\ n_{b.top} - 1 \end{bmatrix} = \begin{bmatrix} 5.043 \\ 7.565 \end{bmatrix} in$

Circular section parameters $\text{if}(SectionType = Circular, \text{"Applicable"}, \text{"N/A"}) = \text{"Applicable"}$

Section diameter: $D := 48 \cdot in$
Cover to hoop bar: $clr := 2 \cdot in$
Number of longitudinal bars: $n_b := 16$

$$\begin{bmatrix} Long\_bar \\ Hoop\_bar \end{bmatrix} := \begin{bmatrix} \#14 \\ \#5 \end{bmatrix}$$

Cross-section area of bars: $\begin{bmatrix} A_{bl} \\ A_{bh} \end{bmatrix} := \overrightarrow{Asb\left(\begin{bmatrix} Long\_bar \\ Hoop\_bar \end{bmatrix}\right)} = \begin{bmatrix} 2.24 \\ 0.31 \end{bmatrix} in^2$

Overall diameter of bars: $\begin{bmatrix} d_{bl} \\ d_{bh} \end{bmatrix} := \overrightarrow{dbo\left(\begin{bmatrix} Long\_bar \\ Hoop\_bar \end{bmatrix}\right)} = \begin{bmatrix} 1.86 \\ 0.7 \end{bmatrix} in$

Bar spacing: $s_{circ} := \dfrac{\pi \cdot (D - 2 \cdot (clr + d_{bh} + 0.5 \cdot d_{bl}))}{n_b} = 8\ in$

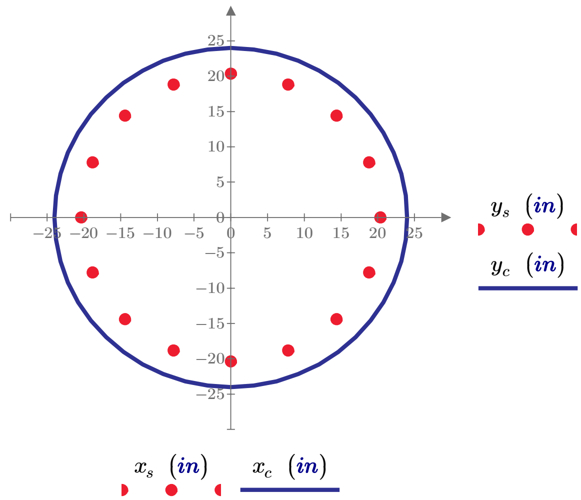
Section properties

Concrete properties:

$$\begin{bmatrix} h \\ A_g \\ I_g \end{bmatrix} := \text{if}\left(SectionType = Circular, \begin{bmatrix} D \\ 0.25 \cdot \pi \cdot D^2 \\ \dfrac{\pi \cdot D^4}{64} \end{bmatrix}, \begin{bmatrix} H \\ B \cdot H \\ \dfrac{B \cdot H^3}{12} \end{bmatrix} \right) = \begin{bmatrix} 48\ in \\ 1809.6\ in^2 \\ 260576.3\ in^4 \end{bmatrix}$$

Total area of reinforcement: $\sum A_s = 35.84\ in^2$

$$n_b = 16 \quad \rho_s := \frac{\sum A_s}{A_g} = 1.98\%$$



Transformed cross-section properties

$$A_t := A_g + \left(\sum A_s\right) \cdot \left(\frac{E_s}{E_c} - 1\right) = 2034\ in^2$$

$$y_t := \frac{A_s \cdot y_s}{A_t} = 0\ in$$

$$I_t := I_g + \left(\frac{E_s}{E_c} - 1\right) \cdot A_s \cdot {y_s}^2 = 307231\ in^4$$

Functions for strain/stress compatibility analysis

x is the distance from the edge in compression

Concrete stress: $fc(c,x) := \text{if}\left(x \le c \cdot \beta_1, \alpha_1 \cdot f'_c, 0 \cdot ksi\right)$
(assumed constant $\alpha_1 \cdot f'_c = 3.4\ ksi$
Section width: $b(x) := \text{if}\left(SectionType = 2, B, 2 \cdot \sqrt{D \cdot x - x^2}\right)$

Resultant of concrete compressive force:

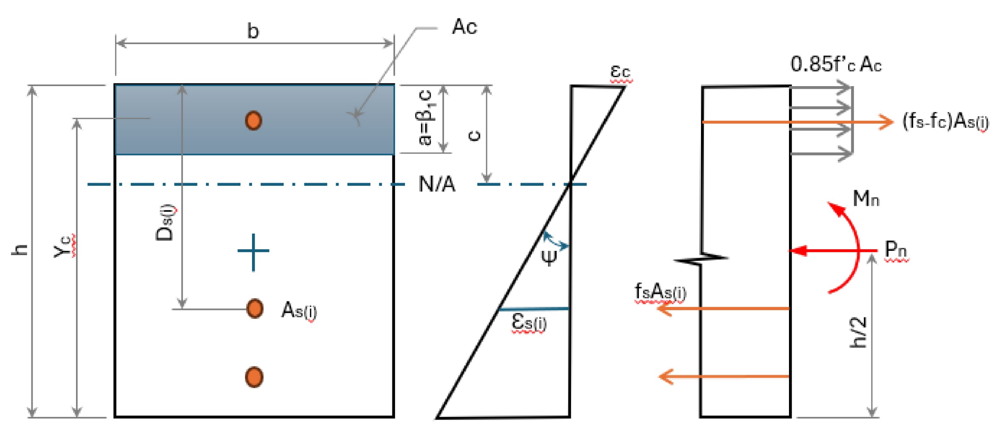
Axial $Pc(c) := \int_{0 \cdot ft}^{min(c \cdot \beta_1, h)} fc(c,x) \cdot b(x)\, dx$ Moment $Mc(c) := \int_{0 \cdot ft}^{min(c \cdot \beta_1, h)} fc(c,x) \cdot b(x) \cdot \left(\frac{h}{2} - x\right) dx$

$$
\begin{aligned}
fs(c, ds) := \;& \text{“Stress in rebar”} \\
& es \leftarrow \varepsilon_{cu} \cdot \left(1 - \frac{ds}{c}\right) \\
& fs \leftarrow \text{sign}(es) \cdot \text{if } |es \cdot E_s| \ge f_y \;\; \| \; f_y \;\; \text{else} \;\; \| \; |es \cdot E_s| \\
& \qquad + \text{if } ds < c \cdot \beta_1 \;\; \| \; fc(c, ds) \;\; \text{else} \;\; \| \; 0 \cdot ksi
\end{aligned}
$$

$$
\begin{aligned}
ds(m) := \;& \text{“returns distance to reinforcing ”} \\
& \text{“bar from compressive edge”} \\
& \text{“m > 0 for positive moment”} \\
& \text{“m < 0 for negative”} \\
& \text{return } 0.5 \cdot h - \text{sign}(m) \cdot y_s
\end{aligned}
$$

$$
\begin{aligned}
\phi f(\varepsilon_t) := \;& \text{if } \varepsilon_t \ge \varepsilon_{tl} \\
& \quad \| \; \phi_t \\
& \text{else if } \varepsilon_t \le \varepsilon_{cl} \\
& \quad \| \; \phi_c \\
& \text{else} \\
& \quad \| \; \phi_c + (\phi_t - \phi_c) \cdot \frac{\varepsilon_t - \varepsilon_{cl}}{\varepsilon_{tl} - \varepsilon_{cl}}
\end{aligned}
$$

$$
\begin{aligned}
PM(c, d_s) := \;& \text{“returns nominal axial and moment”} \\
& \text{“ resistance corresponding to”} \\
& \text{“ location of Neutral Axis – c”} \\
& \text{for } i \in 0 .. n_b - 1 \\
& \quad \| \; f_{s_i} \leftarrow fs\left(c, d_{s_i}\right) \\
& Pn \leftarrow Pc(c) - \sum_{i=0}^{n_b - 1} \left(A_{s_i} \cdot f_{s_i}\right) \\
& Mn \leftarrow Mc(c) + \sum_{i=0}^{n_b - 1} A_{s_i} \cdot f_{s_i} \cdot \left(d_{s_i} - \frac{h}{2}\right) \\
& \text{return } \begin{bmatrix} Pn \\ Mn \end{bmatrix}
\end{aligned}
$$



Nominal resistance

Maximum tension $P_{nt} := -f_y \cdot \sum A_s = -2150.4\ kip$

Maximum compression (5.6.4.4-2, 3)

$$P_{nc} := (0.8 + 0.05 \cdot (SectionType = Circular)) \cdot \left(0.85 \cdot f'_c \cdot \left(A_g - \sum A_s\right) + f_y \cdot \sum A_s\right) = 6953.9\ kip$$

Compute location of neutral axis (c) at control points (positive flexure only):

$d_{s.pos} := ds(1)$ $d_{s.pos.max} := \max\left(d_{s.pos}\right) = 44.37\ in$

Control points: $CP := \begin{bmatrix} \text{"Maximum compression, Pnc"} \\ \text{"Entire section in compression"} \\ \text{"Balance point, fs = fy"} \\ \text{"Compression controlled, es=ecl"} \\ \text{"Tension controlled, es=etl"} \\ \text{"Pure bending, P=0"} \\ \text{"Maximum Tension"} \end{bmatrix}$ $nCP := \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$

Compute location of neutral axis for each control point: ($c.temp0 := 0.9 \cdot h$ $c.temp6 := 0.1 \cdot h$)

$$c.pos := \begin{bmatrix} \mathbf{root}\left(PM\left(c.temp0, d_{s.pos}\right)_0 - P_{nc}, c.temp0\right) \\ h \\ d_{s.pos.max} \cdot \dfrac{\left|\varepsilon_{cu}\right|}{\left|\varepsilon_{cu}\right| + \dfrac{f_y}{E_s}} \\ d_{s.pos.max} \cdot \dfrac{\left|\varepsilon_{cu}\right|}{\left|\varepsilon_{cu}\right| + \varepsilon_{cl}} \\ d_{s.pos.max} \cdot \dfrac{\left|\varepsilon_{cu}\right|}{\left|\varepsilon_{cu}\right| + \varepsilon_{tl}} \\ \mathbf{root}\left(PM\left(c.temp0, d_{s.pos}\right)_0, c.temp0\right) \\ \mathbf{root}\left(PM\left(c.temp6, d_{s.pos}\right)_0 + f_y \cdot \sum A_s, c.temp6\right) \end{bmatrix} = \begin{bmatrix} 48.693 \\ 48 \\ 26.26 \\ 26.622 \\ 16.639 \\ 11.975 \\ 0 \end{bmatrix} in$$

Calculate Axial Force, Bending Moment and steel strain at each control point: $i := 0..6$

$$\begin{bmatrix} P_{cp.pos_i} \\ M_{cp.pos_i} \end{bmatrix} := PM\left(c.pos_i, d_{s.pos}\right) \qquad \varepsilon_{st.pos_i} := \left|\varepsilon_{cu}\right| \cdot \left(\frac{d_{s.pos.max}}{c.pos_i} - 1\right)$$

$$nCP = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad c.pos = \begin{bmatrix} 48.693 \\ 48 \\ 26.26 \\ 26.622 \\ 16.639 \\ 11.975 \\ 0 \end{bmatrix} in \quad P_{cp.pos} = \begin{bmatrix} 6954 \\ 6866 \\ 2940 \\ 3027 \\ 941 \\ 0 \\ -2150 \end{bmatrix} kip \quad M_{cp.pos} = \begin{bmatrix} 1670 \\ 1784 \\ 4454 \\ 4430 \\ 3925 \\ 3097 \\ 0 \end{bmatrix} kip \cdot ft \quad \varepsilon_{st.pos} = \begin{bmatrix} -0.0003 \\ -0.0002 \\ 0.0021 \\ 0.002 \\ 0.005 \\ 0.0081 \\ 1289823.7034 \end{bmatrix}$$

Number of points on interaction diagram $Np := 50$ $j := 0..Np$, bar counter $i := 0..n_b - 1$

Gradually increase distance c from compression edge to neutral axis

$c_j := \text{if}\left(j = 0, 0.01 \cdot in, \frac{2 \cdot h \cdot j}{\beta_1 \cdot Np}\right)$ Curvature $\psi_j := \frac{\varepsilon_{cu}}{c_j}$

Positive moment **Negative moment**

Depth to reinforcement

$d_{s.pos} := ds(1)$

$d_{s.pos.max} := \max(d_{s.pos}) = 44.37\ in$

$d_{s.neg} := ds(-1)$

$d_{s.neg.max} := \max(d_{s.neg}) = 44.37\ in$

Construct nominal P and M interaction diagram using earlier defined function PM2

$\begin{bmatrix} P_{n.pos_j} \\ M_{n.pos_j} \end{bmatrix} := PM\left(c_j, d_{s.pos}\right)$

$\begin{bmatrix} P_{n.neg_j} \\ M_{n.neg_j} \end{bmatrix} := PM\left(c_j, d_{s.neg}\right)$ $M_{n.neg_j} := -M_{n.neg_j}$

Resistance factor per Section 5.6.2.1. Maximum net tensile strain in reinforcing

$\varepsilon_{t.pos_j} := \varepsilon_{cu} \cdot \left(1 - \frac{d_{s.pos.max}}{c_j}\right)$

$\varepsilon_{t.neg_j} := \varepsilon_{cu} \cdot \left(1 - \frac{d_{s.neg.max}}{c_j}\right)$

Resistance factor is interpolated for ε_t between $\varepsilon_{tl} = 0.005$ (0.9) and $\varepsilon_{cl} = 0.002$ (0.75)

$\phi_{pos_j} := \phi f\left(\varepsilon_{t.pos_j}\right)$

$\phi_{neg_j} := \phi f\left(\varepsilon_{t.neg_j}\right)$

Factored resistance

$P_{r.pos_j} := \phi_{pos_j} \cdot min\left(P_{n.pos_j}, j \cdot 10^{-6} \cdot kip + P_{nc}\right)$

$M_{r.pos_j} := \phi_{pos_j} \cdot M_{n.pos_j}$

$P_{r.neg_j} := \phi_{neg_j} \cdot min\left(P_{n.neg_j}, j \cdot 10^{-6} \cdot kip + P_{nc}\right)$

$M_{r.neg_j} := \phi_{neg_j} \cdot M_{n.neg_j}$

Combine results for positive and negative bending in a single array:

$P_n := \text{stack}\left(\text{reverse}\left(P_{n.neg}\right), P_{n.pos}\right)$ $M_n := \text{stack}\left(\text{reverse}\left(M_{n.neg}\right), M_{n.pos}\right)$

$P_r := \text{stack}\left(\text{reverse}\left(P_{r.neg}\right), P_{r.pos}\right)$ $M_r := \text{stack}\left(\text{reverse}\left(M_{r.neg}\right), M_{r.pos}\right)$

$\phi := \text{stack}\left(\text{reverse}\left(\phi_{neg}\right), \phi_{pos}\right)$

Calculate Demand/Capacity Ratios

Acting Loads

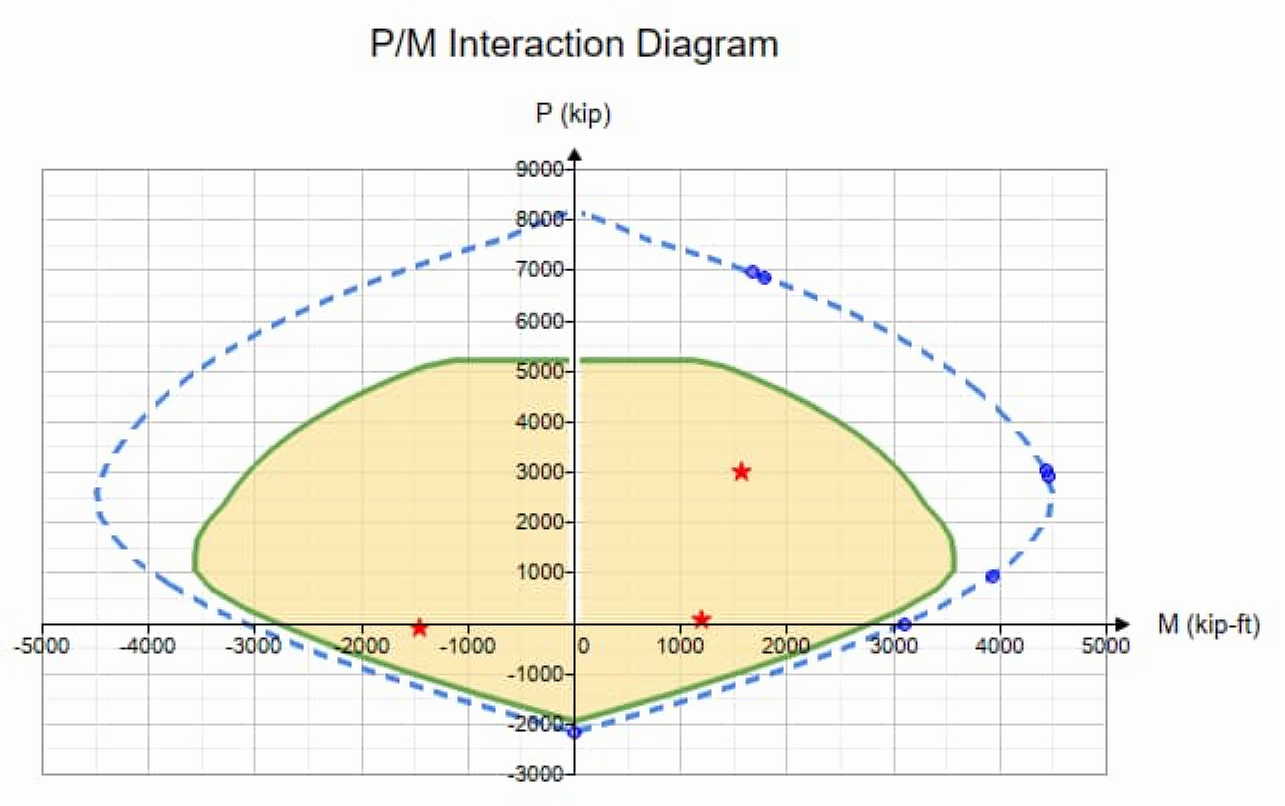
Load_Case	P_u (kip)	M_u $(kip \cdot ft)$
"Str1"	3000	1574
"P max"	70	1200
"P min"	−100	−1448

$$Mr(Pu, Mu) := \text{if } Mu \geq 0 \;\; \left\| \begin{array}{l} \text{"Interpolate Mr for given Pu"} \\ \text{linterp}\left(P_{r.pos}, M_{r.pos}, Pu\right) \end{array} \right. \;\; \text{else} \;\; \left\| \text{linterp}\left(P_{r.neg}, M_{r.neg}, Pu\right) \right.$$

Moment Resistance: $M_{r1} := \overrightarrow{Mr\left(P_u, M_u\right)} = \begin{bmatrix} 3067 \\ 2858 \\ -2677 \end{bmatrix} kip \cdot ft$

Demand/Capacity ratios:

$$DCR_{PM} := \frac{M_u}{M_{r1}} = \begin{bmatrix} 0.513 \\ 0.42 \\ 0.541 \end{bmatrix} \qquad \overrightarrow{Check\left(DCR_{PM} \leq 1\right)} = \begin{bmatrix} \text{"Good"} \\ \text{"Good"} \\ \text{"Good"} \end{bmatrix}$$



Service Load Analysis Functions

To find steel stresses under service load requires to solve two equilibrium equations with two unknowns:
c - location of the neutral axis from compressive face and
f_c - maximum value of triangular concrete compressive stress

Write functions to be later used in the solver block:

Stress in reinforcing bars: $f_s(c, f_c, d_s) := \overrightarrow{f_c \cdot \left(1 - \frac{d_s}{c}\right) \cdot \left(\frac{E_s}{E_c} - (d_s \le c)\right)}$

Describe the concrete area in compression and stress diagram as a function of
x - distance from compression fiber

Section width $b(x) := \mathbf{if}\left(SectionType = Rectangular, B, 2 \cdot \sqrt{D \cdot x - x^2}\right)$

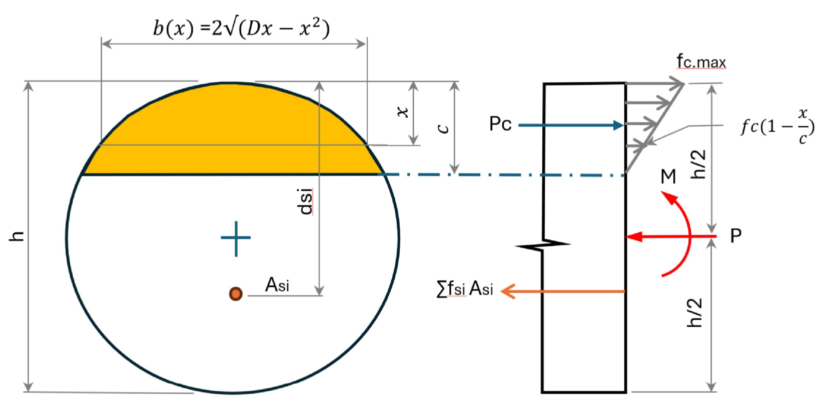
Triangular Concrete stress diagram $fc(x, c, f_c) := f_c \cdot \left(1 - \frac{x}{c}\right)$

Then compressive force in the concrete written as a function of c and fc:

$$P_c(c, f_c) := \int_{0 \cdot ft}^{c} b(x) \cdot fc(x, c, f_c)\, dx$$

Moment about the section's centroid:

$$M_c(c, f_c) := \int_{0 \cdot ft}^{c} b(x) \cdot fc(x, c, f_c) \cdot \left(\frac{h}{2} - x\right) dx$$



Write a function solving equilibrium equations $\sum P = 0$ and $\sum M = 0$ for c_{NA} and $f_{c.max}$ in solver block:

Guess Values

$c_{NA} := 0.33 \cdot h$ $f_{c.max} := -2 \cdot ksi$ (Initial values)

Constraints

$P_c(c_{NA}, f_{c.max}) + A_s \cdot f_s(c_{NA}, f_{c.max}, d_s) = -P$

$M_c(c_{NA}, f_{c.max}) + \overrightarrow{A_s \cdot f_s(c_{NA}, f_{c.max}, d_s)} \cdot \overrightarrow{(0.5 \cdot h - d_s)} = -|M|$

Solver

$cf(P, M, d_s) := \mathbf{Find}(c_{NA}, \mathbf{f_{c.max}})$

Crack Control per AASHTO Section 5.6.7

Service Loads:

$$P := \begin{bmatrix} 200 \\ 100 \\ 0 \end{bmatrix} \cdot kip \qquad M := \begin{bmatrix} 600 \\ -600 \\ 500 \end{bmatrix} \cdot kip \cdot ft$$

Find tensile stresses in outer reinforcing bars (f_{ss}):

Solve for each load: $i := \mathbf{ORIGIN} .. \text{last}(P)$
For M < 0, change ds to h-ds

$$c_{NA_i} := cf\left(P_i, M_i, ds\left(\text{sign}\left(M_i\right)\right)\right)_{\mathbf{ORIGIN}} = \begin{bmatrix} 18.94 \\ 16.38 \\ 14.23 \end{bmatrix} in$$

$$f_{c.max_i} := cf\left(P_i, M_i, ds\left(\text{sign}\left(M_i\right)\right)\right)_{\mathbf{ORIGIN}+1} = \begin{bmatrix} -1.001 \\ -1.02 \\ -0.853 \end{bmatrix} ksi$$

$$f_{ss_i} := f_{c.max_i} \cdot \left(1 - \frac{\max\left(ds\left(\text{sign}\left(M_i\right)\right)\right)}{c_{NA_i}}\right) \cdot \left(\frac{E_s}{E_c} - \left(\max\left(ds\left(\text{sign}\left(M_i\right)\right)\right) \le c_{NA_i}\right)\right) = \begin{bmatrix} 9.77 \\ 12.67 \\ 13.15 \end{bmatrix} ksi$$

Exposure factor: $\gamma_e :=$ Exposure Class: 2 (more severe) $= 0.75$

Distance from tensile reinf. to nearest concrete fiber: $d_{c_i} := h - \max\left(ds\left(M_i\right)\right) = \begin{bmatrix} 3.63 \\ 3.63 \\ 3.63 \end{bmatrix} in$

Spacing of tensile reinforcement: $s_i := \begin{bmatrix} SectionType = 1 \\ M_i \ge 0 \\ M_i < 0 \end{bmatrix} \cdot \begin{bmatrix} s_{circ} \\ s_{bot} \\ s_{top} \end{bmatrix} = \begin{bmatrix} 13.043 \\ 15.564 \\ 13.043 \end{bmatrix} in$

Crack coefficient: $\beta_s := 1 + \frac{d_c}{0.7\ (h - d_c)} = \begin{bmatrix} 1.117 \\ 1.117 \\ 1.117 \end{bmatrix}$ (5.6.7-2)

$$s_{max} := \overrightarrow{\frac{700 \cdot \frac{kip}{in} \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c} = \begin{bmatrix} 40.84 \\ 29.83 \\ 28.48 \end{bmatrix} in$$ (5.6.7-1)

$$\overrightarrow{Check\left(s \le s_{max} \wedge f_{ss} \le 0.6 \cdot f_y\right)} = \begin{bmatrix} \text{“Good”} \\ \text{“Good”} \\ \text{“Good”} \end{bmatrix}$$

Corresponding crack width: $\Delta_{cr} := \overrightarrow{\frac{\left(s + 2 \cdot d_c\right) \cdot \beta_s \cdot f_{ss} \cdot 0.017 \cdot in}{700 \cdot \frac{kip}{in}}} = \begin{bmatrix} 0.14 \\ 0.2 \\ 0.18 \end{bmatrix} mm$

Effective moment of inertia per service loads (per AASHTO Section 5.6.3.5.2)

For earlier defined loads: $P=\begin{bmatrix}200\\100\\0\end{bmatrix}\ kip\quad M=\begin{bmatrix}600\\-600\\500\end{bmatrix}\ kip\cdot ft$

Cracked section properties for rectangular section under service load:

$$A_{cr_i}:=B\cdot c_{NA_i}+A_s\cdot\overrightarrow{\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)}=\begin{bmatrix}927.041\\834.705\\761.655\end{bmatrix}\ in^2$$

$$y_{cr_i}:=\frac{0.5\cdot B\cdot c_{NA_i}^{\ 2}+\overrightarrow{A_s\cdot d_s}\cdot\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)}{A_{cr_i}}=\begin{bmatrix}13.56\\13.106\\12.9\end{bmatrix}\ in$$

$$I_{cr_i}:=\frac{B\cdot c_{NA_i}^{\ 3}}{12}+B\cdot c_{NA_i}\cdot\left(y_{cr_i}-0.5\cdot c_{NA_i}\right)^2+\overrightarrow{A_s\cdot\left(y_{cr_i}-d_s\right)^2}\cdot\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)=\begin{bmatrix}113663\\111887\\111493\end{bmatrix}\ in^4$$

$$\frac{I_{cr}}{I_g}=\begin{bmatrix}0.436\\0.429\\0.428\end{bmatrix}$$

Concrete modulus of rupture per 5.4.2.6 $\quad f_r:=0.24\cdot\sqrt{f'_c\cdot ksi}=0.48\ ksi$

Cracking moment $\quad M_{cr}:=\left(f_r+\frac{P}{A_g}\right)\cdot\frac{I_g}{0.5\cdot h}=\begin{bmatrix}534.29\\484.29\\434.29\end{bmatrix}\ kip\cdot ft$ (5.6.3.5.2-1 modified)

Effective moment of inertia:

$$I_{e_i}:=min\left(\left(\frac{M_{cr_i}}{\left|M_i\right|}\right)^3\cdot I_g+\left(1-\left(\frac{M_{cr_i}}{\left|M_i\right|}\right)^3\right)\cdot I_{cr_i},I_g\right)=\begin{bmatrix}217403\\190077\\209188\end{bmatrix}\ in^4\quad\text{(5.6.3.5.2-1)}$$

$$\frac{I_e}{I_g}=\begin{bmatrix}0.834\\0.729\\0.803\end{bmatrix}$$

Design for Shear and Torsion

Loads

$P_u := 300 \cdot kip$

$V_u := 300 \cdot kip$

$M_u := 1000 \cdot kip \cdot ft$

$T_u := 500 \cdot kip \cdot ft$

Resistance factors

$\phi_f := 0.9$

$\phi_v := 0.9$

Transverse reinforcing

Yield strength, $f_{yh} := 60 \cdot ksi$

Spacing of transverse reinforcing, $s := 6 \cdot in$

$Hoop\_bar = 5$

$A_{bh} = 0.31\ in^2$

Number of hoops in the bundle, $n_h := 1$

For rectangular concrete cross-section:

Number of links in the direction of shear force:

$n_v := 2$

$Link\_bar := \#5$

$A_{vb} := Asb(Link\_bar) = 0.31\ in^2$

Torsional properties

$$\begin{bmatrix} p_c \\ A_{cp} \\ p_h \\ A_{oh} \\ A_o \end{bmatrix} := \left\| \begin{array}{l} \text{if } SectionType = 1 \\ \quad p_c \leftarrow \pi \cdot D \\ \quad A_{cp} \leftarrow 0.25 \cdot \pi \cdot D^2 \\ \quad c_b \leftarrow clr + 0.5 \cdot d_{bh} \\ \quad p_h \leftarrow \pi \cdot (D - 2 \cdot c_b) \\ \quad A_{oh} \leftarrow 0.25 \cdot \pi \cdot (D - 2 \cdot c_b)^2 \\ \quad b_e \leftarrow \dfrac{A_{cp}}{p_c} \\ \quad A_o \leftarrow 0.25 \cdot \pi \cdot (D - b_e)^2 \\ \text{else} \\ \quad p_c \leftarrow 2 \cdot (B + H) \\ \quad A_{cp} \leftarrow B \cdot H \\ \quad c_b \leftarrow clr_{tb} + d_{tb} \\ \quad p_h \leftarrow p_c - 8 \cdot c_b \\ \quad A_{oh} \leftarrow (B - 2 \cdot c_b) \cdot (H - 2 \cdot c_b) \\ \quad b_e \leftarrow \dfrac{A_{cp}}{p_c} \\ \quad A_o \leftarrow (H - b_e) \cdot (B - b_e) \\ \text{return stack}(p_c, A_{cp}, p_h, A_{oh}, A_o) \end{array} \right.$$

$$\begin{bmatrix} p_c \\ p_h \end{bmatrix} = \begin{bmatrix} 150.8 \\ 136.03 \end{bmatrix} in$$

$$\begin{bmatrix} A_{cp} \\ A_{oh} \\ A_o \end{bmatrix} = \begin{bmatrix} 1809.6 \\ 1472.5 \\ 1017.9 \end{bmatrix} in^2$$

Area of torsional reinforcing $A_t := A_{bh} \cdot n_h = 0.31\ in^2$

Area of shear reinforcing

$A_v := \text{if}(SectionType = 1, 2 \cdot n_h \cdot A_{bh}, 2\ A_{bh} + n_v \cdot A_{vb}) = 0.62\ in^2$

Area of longitudinal reinforcing on flexural tension side $A_{st} := A_s \cdot (d_s \geq 0.5 \cdot h) = 20.16\ in^2$

Area of concrete on flexural tension side $A_{ct} := 0.5 \cdot A_g = 904.779\ in^2$

Effective depth for flexure $d_e := \dfrac{\overrightarrow{A_s \cdot (d_s \geq 0.5 \cdot h) \cdot d_s}}{A_{st}} = 35.379\ in$

Location of neutral axis $cna := \text{linterp}(\text{if}(M_u \geq 0, P_{r.pos}, P_{r.neg}), c, P_u) = 13.711\ in$

Effective depth for shear $d_v := \max\left(\begin{bmatrix} d_e - 0.5 \cdot cna \cdot \beta_1 \\ 0.9 \cdot d_e \\ 0.72 \cdot h \end{bmatrix}\right) = 34.56\ in$

Effective width for shear $b_v := \text{if}(SectionType = 1, D, B) = 48\ in$

Minimum bending moment to consider $M_u := \max(|M_u|, V_u \cdot d_v) = 1000\ kip \cdot ft$

Consideration of torsion

$K := min\left(\sqrt{1+\frac{P_u}{A_g \cdot 0.126 \cdot \sqrt{f'_c \cdot ksi}}}, 2\right) = 1.288$ (5.7.2.1-6)

$T_{cr} := 0.126 \cdot K \cdot \sqrt{f'_c \cdot ksi} \cdot \frac{{A_{cp}}^2}{p_c} = 587.2\ kip \cdot ft$ (5.7.2.1-4)

$T_u = 500\ kip \cdot ft$

$Consider\_torsion? := (T_u > 0.25 \cdot \phi_v \cdot T_{cr}) = 1$ (5.7.2.1-3)

Calculate β and θ

$V_{eff} := \sqrt{{V_u}^2 + Consider\_torsion? \cdot \left(\frac{0.9 \cdot p_h \cdot T_u}{2 \cdot A_o}\right)^2} = 469.255\ kip$ (5.7.3.4.2-5)

Concrete shear stress $v_u := \frac{V_{eff}}{\phi_v \cdot b_v \cdot d_v} = 0.314\ ksi$ $\frac{v_u}{f'_c} = 0.079$

Tensile strain at centroid of A_{st}

$$\varepsilon_s := \frac{\frac{M_u}{d_v} - 0.5 \cdot P_u + V_{eff}}{E_s \cdot A_{st} + E_c \cdot A_{ct} \cdot \left(\frac{M_u}{d_v} - 0.5 \cdot P_u + V_{eff} < 0\right)} = 0.001$$ (5.7.3.4.2-4)

$\varepsilon_s := \max(\varepsilon_s, -0.0004) = 0.001$

$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_s} = 2.588$ (5.7.3.4.2-1)

$\theta := (29 + 3500 \cdot \varepsilon_s) \cdot deg = 32.99\ deg$ (5.7.3.4.2-3)

Concrete shear resistance $V_c := 0.0316 \cdot \beta \cdot \sqrt{f'_c \cdot ksi} \cdot b_v \cdot d_v = 271.3\ kip$

Minimum shear reinforcement requirements

$Shear\_reinf\_required := (V_u > 0.5 \cdot \phi_v \cdot V_c) = 1$

$A_{v.min} := 0.0316 \cdot \sqrt{f'_c \cdot ksi} \cdot \frac{b_v \cdot s}{f_y} \cdot Shear\_reinf\_required = 0.303\ in^2$ 5.7.2.5-1

$A_v = 0.62\ in^2$ $Check\ (A_v \geq A_{v.min}) = \text{"Good"}$

Check maximum allowable spacing of shear reinforcement

$s_{max} := \text{if}\ (v_u < 0.125 \cdot f'_c, min(0.8 \cdot d_v, 24 \cdot in), min(0.4 \cdot d_v, 12 \cdot in)) = 24\ in$

$s = 6\ in$ $Check\ (s \leq s_{max}) = \text{"Good"}$

Torsion resistance

$$T_r := \phi_v \cdot \frac{2 \cdot A_o \cdot A_t \cdot f_{yh} \cdot \cot(\theta)}{s} = 729.1\ kip \cdot ft$$ (5.7.3.6.2-1)

Demand/capacity ratio $DCR_{torsion} := \frac{T_u}{T_r} = 0.686$ $Check\left(\frac{T_u}{T_r} \le 1\right) = \text{"Good"}$

Shear resistance

Calculate hoop reinforcement required for torsion

$$A_{t.req'd} := Consider\_torsion? \cdot \frac{T_u}{T_r} \cdot A_t = 0.213\ in^2$$

Shear reinforcement after reduction for torsion

$$A_v := A_v - A_t \cdot \text{if}\left(SectionType = 1, \frac{\pi}{4}, 1\right) = 0.377\ in^2$$

Steel shear resistance $V_s := \frac{A_v \cdot f_{yh} \cdot d_v \cdot \cot(\theta)}{s} = 200.5\ kip$ (5.7.3.3-4)

Maximum shear resistance $V_{n.max} := 0.25 \cdot f'_c \cdot b_v \cdot d_v = 1658.9\ kip$ (5.7.3.3-2)

Factored resistance $V_r := \phi_v \cdot min\left(V_c + V_s, V_{n.max}\right) = 424.6\ kip$ (5.7.3.3-1, 2)

Demand/capacity ratio $DCR_{shear} := \frac{V_u}{V_r} = 0.707$ $Check\left(\frac{V_u}{V_r} \le 1\right) = \text{"Good"}$

Longitudinal reinforcement requirements - 5.7.3.5 and 5.7.3.6.3

Areas of longitudinal reinforcement required for:

a) Flexure and axial force (1st half of (5.7.3.5-1) $A_{sf} := \frac{\left(\frac{M_u}{\phi_f \cdot d_v} + \frac{0.5 \cdot -P_u}{\phi_f}\right)}{f_y} = 3.652\ in^2$

b) Shear (2nd half of (5.7.3.5-1)) $A_{sv} := \frac{\left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s\right) \cdot \cot(\theta)}{f_y} = 5.985\ in^2$

c) Torsion (distributed around perimeter, based on (5.7.3.6.3-1))

$$A_l := \left(\frac{0.45 \cdot p_h \cdot T_u}{2 \cdot A_o \cdot \phi_v}\right) \cdot \frac{\cot(\theta)}{f_y} = 5.147\ in^2$$

Total longitudinal reinforcement req'd on the flexural tension side of the member (5.7.3.6.3-1)

$$A_{st.req'd} := A_{sf} + \sqrt{{A_{sv}}^2 + {A_l}^2} = 11.546\ in^2$$

Provided reinforcement $A_{st} = 20.16\ in^2$

Demand/capacity ratio $DCR_{long.reinf} := \frac{A_{st.req'd}}{A_{st}} = 0.573$ $Check\left(\frac{A_{st.req'd}}{A_{st}} \le 1\right) = \text{"Good"}$